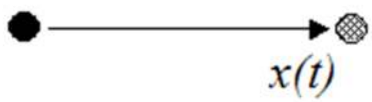
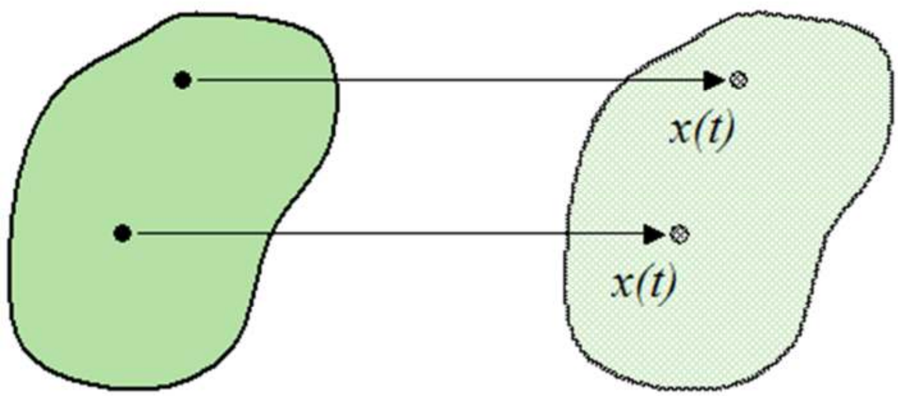
# Rectilinear Motion

Rectilinear motion is another name for straight-line motion. This type of motion describes the movement of a particle or a body.

A body is said to experience rectilinear motion if any two particles of the body travel the same distance along two parallel straight lines. The figures below illustrate rectilinear motion for a particle and body.




Rectilinear motion for a body:




In the above figures, *x(t)* represents the position of the particles along the direction of motion, as a function of time *t*.

Given the position of the particles, *x(t)*, we can calculate the displacement, velocity, and acceleration. These are important quantities to consider when evaluating the kinematics of a problem.

A common assumption, which applies to numerous problems involving rectilinear motion, is that acceleration is constant. With acceleration as constant we can derive equations for the position, displacement, and velocity of a particle, or body experiencing rectilinear motion.

The easiest way to derive these equations is by using Calculus.

The acceleration is given by

$$\frac{d^2x}{dt^2} = a$$

where $a$ is the acceleration, which we define as constant.

Integrate the above equation with respect to time, to obtain velocity. This gives us

$$v(t) = \int a\,dt = C_1 + at$$

where $v(t)$ is the velocity and $C_1$ is a constant.

Integrate the above equation with respect to time, to obtain position. This gives us

$$x(t) = \int v(t)\,dt = C_2 + C_1 t + \frac{1}{2}at^2$$

where $x(t)$ is the position and $C_2$ is a constant.

The constants $C_1$ and $C_2$ are determined by the initial conditions at time $t$ = 0. The initial conditions are:

At time $t$ = 0 the position is $x_1$.

At time $t$ = 0 the velocity is $v_1$.

Substituting these two initial conditions into the above two equations we get

$$v(0) = v_1 = C_1$$

$$x(0) = x_1 = C_2$$

Therefore $C_1 = v_1$ and $C_2 = x_1$.

This gives us

$$x(t) = x_1 + v_1 t + \frac{1}{2}at^2$$

$$v(t) = v_1 + at$$

For convenience, set $x(t) = x_2$ and $v(t) = v_2$. As a result

$$x_2 = x_1 + v_1 t + \frac{1}{2}at^2 \quad \text{(1) – position equation}$$

$$v_2 = v_1 + at \quad \text{(2) – velocity equation}$$

Displacement is defined as $\Delta d = x_2 — x_1$. Therefore, equation (1) becomes

$$\Delta d = v_1 t + \frac{1}{2}at^2 \quad \text{(3) – displacement equation}$$

If we wish to find an equation that doesn't involve time *t* we can combine equations (2) and (3) to eliminate time as a variable. This gives us

$$v_2^2 = v_1^2 + 2a(\Delta d) \quad \text{(4) – 2}^{\text{nd}}\text{ velocity equation}$$

Equations (1), (2), (3), and (4) fully describe the motion of particles, or bodies experiencing rectilinear (straight-line) motion, where acceleration *a* is constant.

**With CONSTANT ACCELERATION!**

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**WHEN ACCELERATION IS NOT CONSTANT!**

For the cases where acceleration is not constant, new expressions have to be derived for the position, displacement, and velocity of a particle. If the acceleration is known as a function of time, we can use Calculus to find the position, displacement, and velocity, in the same manner as before.

Alternatively, if we are given the position *x(t)* as a function of time, we determine the velocity by differentiating *x(t)* once, and we determine the acceleration by differentiating *x(t)* twice.

For example, let's say the position *x(t)* of a particle is given by

$$x(t) = \cos(2t) + 4t^3$$

Thus, the velocity *v(t)* is given by

$$v(t) = \frac{dx}{dt} = -2\sin(2t) + 12t^2$$

The acceleration *a(t)* is given by

$$a(t) = \frac{d^2x}{dt^2} = -4\cos(2t) + 24t$$